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DRAFT: NUMERICAL FLIGHT VEHICLE FORWARD DYNAMICS WITH STATE-SPACE LIE-GROUP INTEGRATION SCHEME

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ABSTRACT

Dynamic simulation procedures of flight vehicle maneuvers need robust and efficient integration methods in order to allow for reliable simulation missions. Derivation of such integration schemes in Lie-group settings is especially efficient since the coordinate-free Lie-group dynamical models operate directly on SO(3) rotational matrices and angular velocities, avoiding local rotation parameters and artificial algebraic constraints as well as kinematical differential equations. In the adopted modeling approach, a state-space of the flight vehicle (modeled as a multi-body system comprising rigid bodies) is modeled as a Lie-group. The numerical algorithm is demonstrated and tested within the framework of characteristic case study the several case studies of the aircraft 3D maneuvers.

NOMENCLATURE

C_D, C_Y, C_L Drag, side-force and lift aerodynamic coefficients.
C_x, C_y, C_z Force aerodynamic coefficients in body frame.
C_l, C_m, C_n Moment aerodynamic coefficients in body frame.
b, c_A Reference dimensions for aerodynamic moments, wing span and mean aerodynamic chord, respectively.
h Altitude.
$\mathbf{M}$ Inertia matrix.
P_{eng} Engine power available.
p Position and attitude vector.
p, q, r Elements of angular velocity, in body frame ω.
p^*, q^*, r^* Non dimensional angular velocities p, q, r.
$\mathbf{Q}$ Forces and moments vector.
$\mathbf{Q}_f^b$ Applied, aerodynamic force vector in body frame.
$\mathbf{Q}_m^b$ Applied, aerodynamic moment vector in body frame.
$\mathbf{Q}_p^b$ Applied force vector from propulsion group in body frame.
$\mathbf{R}$ Rotation matrix from body to global inertial frame.
$\mathbf{R}_v$ Rotation matrix from body to aerodynamic velocity frame; rotation about y and z axes for α and β respectively.
S_{ref} Reference area.
T_a Available thrust.
u, v, w Elements of velocity vector in body frame, $\mathbf{v}^b$.
$\mathbf{v}$ Velocity vector in global frame.
V Aerodynamic velocity.
$\mathbf{x}$ Position vector in global frame.
α Angle of attack.
α_T Inclination of the engine to the longitudinal axis of body frame.
β Sideslip angle.
δ_l, δ_m, δ_n Aerodynamic controls, angle of deflection of control surfaces: aileron, elevator and rudder.
η_{prop} Propeller efficiency.

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ϕ, θ, ψ Attitude of the aircraft: roll, pitch and yaw angles, elements of Ψ.

Ψ Attitude vector.

ρ Density.

ω Angular velocity vector.

$()_0$ Initial conditions at $t = 0$.

$()^b$ Vector components in body frame.

INTRODUCTION

Dynamic simulation procedures of the flight vehicle (rotorcraft, UAV, satellite) 3D maneuvers need robust and efficient integration methods in order to allow for reliable, and possibly real-time, simulation missions. Derivation of such integration schemes in Lie-group settings is especially efficient since the coordinate-free Lie-group dynamical models operate directly on $SO(3)$ rotational matrices and angular velocities, avoiding local rotation parameters and artificial algebraic constraints as well as kinematical differential equations.

THEORETICAL FORMULATION

In the adopted modeling approach, a configuration space of the flight vehicle (modeled as a multibody system (MBS) comprising k rigid bodies) is modeled as a Lie-group $G = \mathscr{R}^3 \times ... \times \mathscr{R}^3 \times SO(3) \times ... \times SO(3)$ (k copies of $\mathscr{R}^3 \times SO(3)$) with the elements of the form $p = (\mathbf{x}_1, ..., \mathbf{x}_k, \mathbf{R}_1, ..., \mathbf{R}_k)$. Each factor $\mathscr{R}^3 \times SO(3)$ represents configuration of the one single flight vehicle body, represented by $(\mathbf{x}_i, \mathbf{R}_i)$ - its position vector and the rotation matrix w.r.t. a global frame. The angular velocity of a body is given by the left-invariant vector field $\widetilde{\omega}_i \in so(3)$ defined as $\dot{\mathbf{R}}_i(t) = \mathbf{R}_i(t)\widetilde{\omega}_i$ with $so(3)$ being the Lie algebra of $SO(3)$. A velocity of the one flight vehicle body (body i) can thus be represented by the couple $(\mathbf{v}_i, \omega_i) \in \mathscr{R}^3 \times so(3)$.

Aiming on the application of the Lie-group integration scheme proposed in [1], also the flight vehicle state space must be expressed as a Lie-group. Therefore, the state space $\mathscr{S} = \mathscr{R}^3 \times ... \times \mathscr{R}^3 \times SO(3) \times ... \times SO(3) \times \mathscr{R}^3 \times ... \times \mathscr{R}^3 \times so(3) \times ... \times so(3) \widetilde{=} TG$ is introduced, with the elements $q = (\mathbf{x}_1, ..., \mathbf{x}_k, \mathbf{R}_1, ..., \mathbf{R}_k, \mathbf{v}_1, ..., \mathbf{v}_k, \omega_1, ..., \omega_k)$. This is a Lie-group itself and possess the Lie-algebra $\mathscr{s} = \mathscr{R}^3 \times ... \times \mathscr{R}^3 \times so(3) \times ... \times so(3) \times \mathscr{R}^3 \times ... \times \mathscr{R}^3 \times \mathscr{R}^3 \times ... \times \mathscr{R}^3$ with the element $z = (\mathbf{v}_1, ..., \mathbf{v}_k, \widetilde{\omega_1}, ..., \widetilde{\omega_k}, \dot{\mathbf{v}}_1, ..., \dot{\mathbf{v}}_k, \dot{\omega}_1, ..., \dot{\omega}_k)$. Furthermore, the operations on the Lie-group $\mathscr{S}$ and its Lie-algebra $\mathscr{s}$ (such as product in $\mathscr{R}^3 \times SO(3)$), addition in $\mathscr{R}^3 \times so(3)$, multiplication by scalar in $\mathscr{R}^3 \times so(3)$, exponential map in $\mathscr{R}^3 \times so(3)$ and bracket in $\mathscr{R}^3 \times so(3)$ can be introduced [1], allowing synthesis of the subsequent integration routines. To formulate flight vehicle dynamical model in the introduced state space, the MBS constrained Boltzmann-Hamel equations are shaped in the form

$$\begin{aligned} \mathbf{M}(p)\dot{\mathbf{v}} + \mathbf{C}^T(p)\lambda &= \mathbf{Q}(p, \mathbf{v}, t) \\ \dot{p} &= p \cdot \widetilde{\mathbf{v}} \\ \Phi(p) &= 0, \end{aligned} \tag{1}$$

where $\mathbf{M}$ is $n \times n$ dimensional generalized inertia matrix, $\mathbf{v} \in \mathscr{R}^n$, $\mathbf{v} = [\mathbf{v}_1, ..., \mathbf{v}_k, \omega_1, ..., \omega_k]^T$ are the system velocities (k bodies are assumed), $\mathbf{Q}$ represents the external and all other forces, $\lambda \in \mathscr{R}^m$ is the vector of Lagrange multipliers and $\mathbf{C}$ is $m \times n$ dimensional constraint Jacobian, such that

$$\Phi'(p) \cdot \widetilde{\mathbf{v}} = \mathbf{C}(p)\mathbf{v},$$

where Φ' is the differential mapping of the constraint mapping $\Phi : G \to \mathscr{R}^m$. Consequently, a system is constrained to evolve on the n-m dimensional sub-manifold $S = \{p \in G : \Phi(p) = 0\}$. During numerical integration, a flight vehicle 3D motion can be numerically reconstructed from the velocity field $\mathbf{v}$, by using the equation $\dot{p} = p \cdot \widetilde{\mathbf{v}}$ in (1) [1], [2]. The system (1) is a DAE system of index 3. Within the framework of the paper, the equation (1) will be re-shaped into the DAE of index 1 form by including the kinematical constraints at the acceleration level

$$\ddot{\Phi}(p, \mathbf{v}, \dot{\mathbf{v}}) = 0$$

(instead of $\Phi(p) = 0$) and integrated by the integration algorithms based on the state space formulation [1]. During integration, DAE hidden constraints will be stabilized via Lie-group stabilization algorithms described in [1] and [3].

CASE STUDY

For the prime case study a simple motion of a general aviation airplane, modeled as a single 6DOF rigid body problem, is selected. Simple predefined controls are implemented with a resulting turn with altitude loss.

For global inertial frame a common frame in flight mechanics is selected: North-East-Down (NED) coordinate system and it is applied for the description of trajectory $\mathbf{x}$. Its base point is selected to be a starting point on the ground level.

Description of general aviation airplane model

Main data of the general aviation airplane example applied in this case study is presented in the Tab. 1 according to the [4] and [5].

Presented model of general aviation aircraft includes full linear aerodynamic model. Implemented aerodynamic coefficients

TABLE 1. BASIC DATA OF GENERAL AVIATION AIRPLANE EXAMPLE.

S_{ref}	15.1 m^2	b	8.77 m
c_A	1.698 m	α_T	3.3°
V_0	45 m/s	h_0	500 m
m_0	1088 kg	I_x	1450 kg m^2
I_y	1693 kg m^2	I_z	3134 kg m^2

are defined in aerodynamic velocity frame with a base point in mass center, for forces drag, side force and lift:

$$\begin{aligned} C_D &= C_{D_0} + K C_L^2 \\ C_Y &= C_{Y_\beta}\beta + C_{Y_p} p^* + C_{Y_r} r^* + C_{Y_{\delta_n}}\delta_n \\ C_L &= C_{L_0} + C_{L_\alpha}\alpha + C_{L_{\dot\alpha}}\dot\alpha^* + C_{L_q} q^* + C_{L_{\delta_m}}\delta_m \,, \end{aligned} \tag{2}$$

and for moments roll, pitch and yaw:

$$\begin{aligned} C_l &= C_{l_\beta}\beta + C_{l_p} p^* + C_{l_r} r^* + C_{l_{\delta_l}}\delta_l + C_{l_{\delta_n}}\delta_n \\ C_m &= C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\dot\alpha}}\dot\alpha + C_{m_q} q^* + C_{m_{\delta_m}}\delta_m \\ C_n &= C_{n_\beta}\beta + C_{n_p} p^* + C_{n_r} r^* + C_{n_{\delta_l}}\delta_l + C_{n_{\delta_n}}\delta_n \,. \end{aligned} \tag{3}$$

Force aerodynamic coefficients are transformed to the body frame

$$\begin{bmatrix} C_x \\ C_y \\ C_z \end{bmatrix} = \mathbf{R}_v^T \cdot \begin{bmatrix} -C_D \\ C_Y \\ -C_L \end{bmatrix} .$$

All aerodynamic gradients in (2), (3) are constant with respect to Mach number variation for this particular aircraft. Resulting aerodynamic force in body frame is

$$\mathbf{Q}_f^b = \frac{1}{2}\rho V^2 S_{ref} \begin{bmatrix} C_x \\ C_y \\ C_z \end{bmatrix} ,$$

and moment

$$\mathbf{Q}_m^b = \frac{1}{2}\rho V^2 S_{ref} \begin{bmatrix} b\cdot C_l \\ c_A \cdot C_m \\ b \cdot C_n \end{bmatrix} .$$

For the analyzed small propeller aircraft a model of piston engine is applied with available power P_{eng} as a function of control δ_{th}, pressure and temperature [5]. Available thrust force at the propeller is $T_a = \frac{\eta_{prop} P_{eng}}{V}$ and associated force in body frame $\mathbf{Q}_p^b = [T_a \cos\alpha_T \quad 0 \quad T_a \sin\alpha_T]^T$. It is assumed that thrust is directed through the aircraft's mass center and additional moments of the propulsion due to the nonsymetrical flow at the propeller are neglected. Complete applied force in global frame is

$$\mathbf{Q}_{force} = \mathbf{R}\left(\mathbf{Q}_f^b + \mathbf{Q}_p^b\right) .$$

For this case study a constant mass is assumed, following which there is no change in any inertial element during the presented simulation.

Controls available for presented model of aircraft presents deflection of aerodynamic control surfaces δ_l, δ_m, δ_n, respectively ailerons, elevator and rudder, and which are included in aerodynamic coefficients (2), (3). Furthermore there is a thrust command δ_{th}.

Results of simulation

For the presented simulation a predefined controls are applied as presented on Fig. 1, while thrust command was kept constant at 90% of maximum available in given conditions.

Initial conditions of simulation were determined from the trim: equilibrium of forces along vertical axis of velocity frame for horizontal flight and zero pitching moment, for given aerodynamic velocity V_0 and altitude h_0. Complete initial conditions at

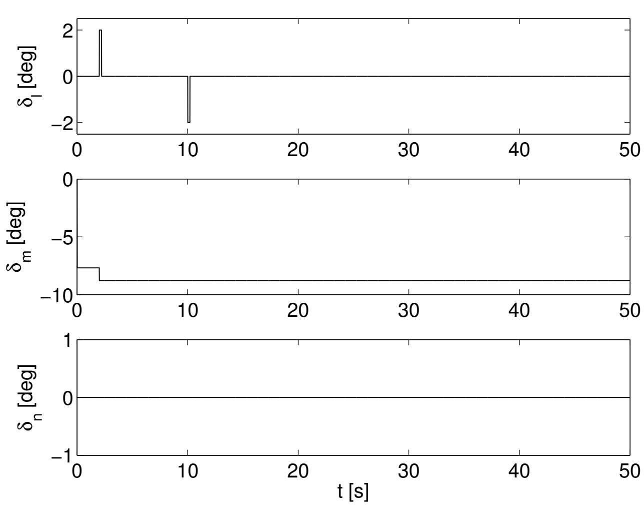

FIGURE 1. TIME VARIATION OF AIRPLANE'S AERODYNAMIC CONTROLS: AILERON, ELEVATOR AND RUDDER.

$t = 0$ are:

$$p = \begin{bmatrix} \mathbf{x} \\ \Psi \end{bmatrix} = \begin{bmatrix} (x_1)_0 \\ (x_2)_0 \\ (x_3)_0 \\ \phi_0 \\ \theta_0 \\ \psi_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -h_0 \\ 0 \\ \alpha_0 \\ 0 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} \mathbf{v} \\ \omega \end{bmatrix} = \begin{bmatrix} \mathbf{R}_0(\mathbf{R}_v)_0^T \begin{bmatrix} V_0 \\ 0 \\ 0 \end{bmatrix} \\ p_0 \\ q_0 \\ r_0 \end{bmatrix} = \begin{bmatrix} \mathbf{R}_0 \begin{bmatrix} V_0 \cos\beta_0 \cos\alpha_0 \\ V_0 \sin\beta_0 \\ V_0 \cos\beta_0 \sin\alpha_0 \end{bmatrix} \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

For described simulation it is assumed that there is no wind, subsequently aerodynamic velocity equals velocity of body frame with respect to global. For discussed airplane and described initial conditions it was set: $\alpha_0 = 4.6°$, $\beta_0 = 0°$, $\delta_{m_0} = 7.7°$.

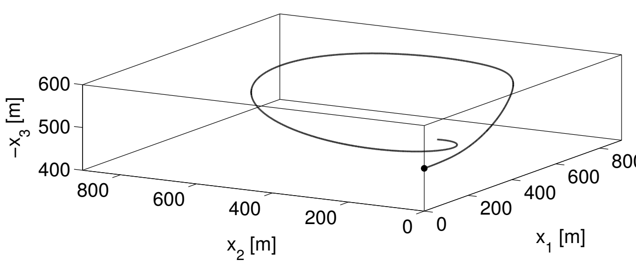

FIGURE 2. TRAJECTORY OF AIRPLANE'S MASS CENTER.

Resulting 3D trajectory of the airplane's mass center is presented on Fig. 2, time variations of $\mathbf{x}$ components on Fig. 3 and its ground projection on Fig. 4 [1]. Following the ailerons deflection at $t = 2$ s, aircraft starts turning. Elevator is set to the trim value at the start of simulation. Since there is no elements of trim implemented in model, airplane eventually accelerates and looses altitude.

Velocities in flight mechanics are commonly presented in the body frame

$$\mathbf{v}^b = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \mathbf{R}^T \mathbf{v},$$

as on Fig. 5. Angular velocities, also in body frame, are given on Fig. 6. At $t = 2$ s and $t = 10$ s an ailerons deflection $\delta_l = 2°$ is

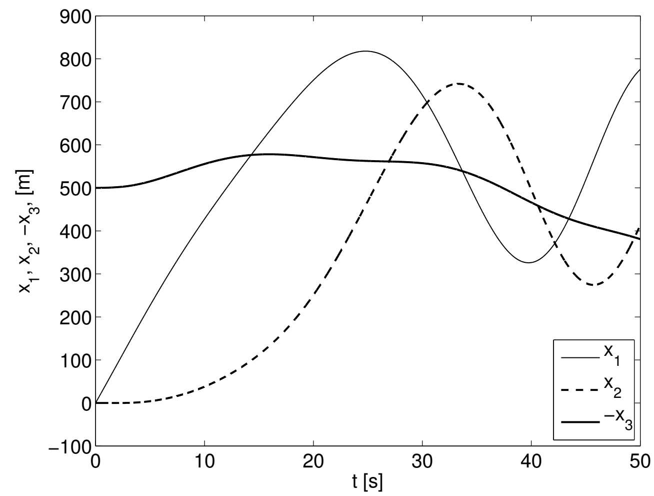

FIGURE 3. TIME VARIATION OF THE AIRPLANE'S MASS CENTER COORDINATES IN GLOBAL FRAME.

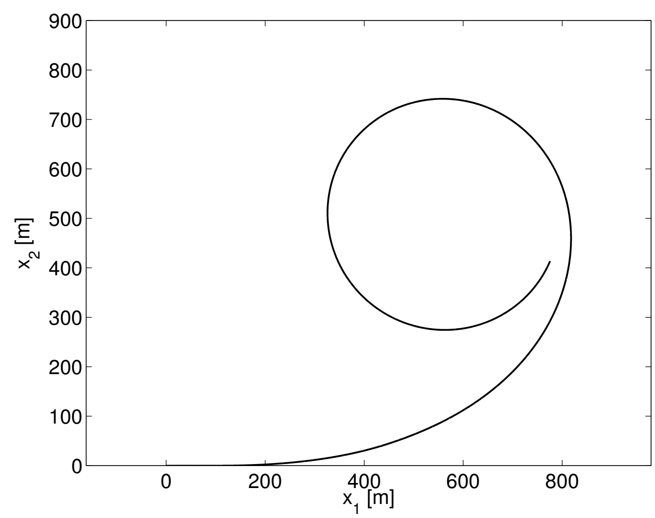

FIGURE 4. GROUND PROJECTION OF TIME VARIATION OF AIRPLANE'S MASS CENTER IN GLOBAL FRAME.

applied what is followed by quick response of the aircraft with rotation about longitudinal axis, roll p, as presented on Fig. 6.

Rotation in flight mechanics, attitude of the airplane, is commonly represented with De'Sparre angles (as in [5] what is in agreement with [6])

$$\Psi(t) = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix},$$

[1] In the selected global inertial frame – NED: altitude above ground is $h = -x_3$.

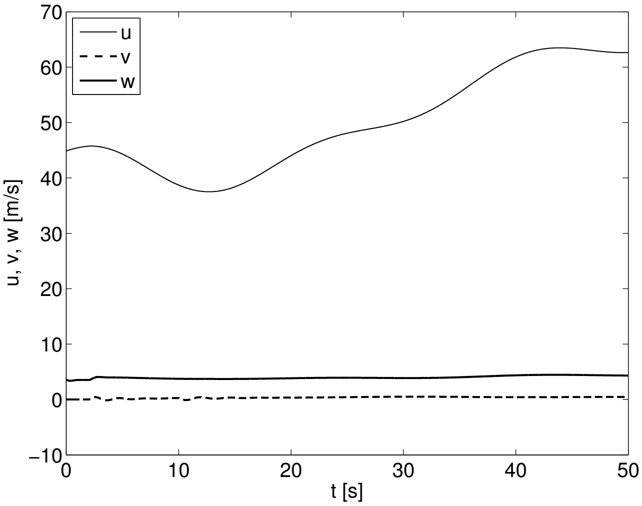

FIGURE 5. COMPONENTS OF VELOCITY VECTOR IN BODY FRAME $\mathbf{v}^b$.

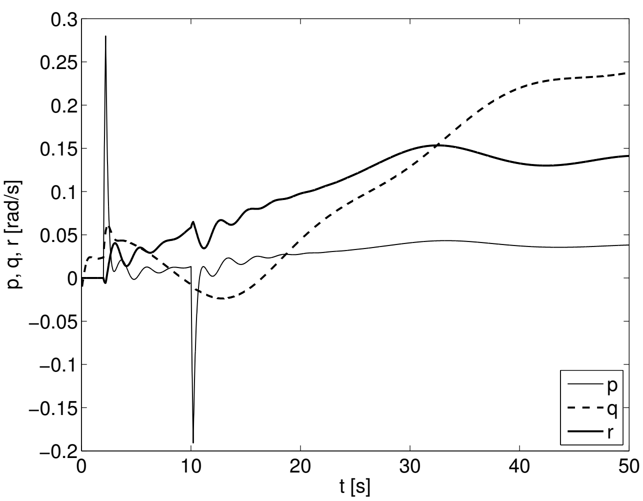

FIGURE 6. COMPONENTS OF ANGULAR VELOCITY IN BODY FRAME ω: ROLL, PITCH AND YAW.

which are also referred as Euler angles. This is representation of time variation of rotation matrix $\mathbf{R}(t)$. In presented case study time variation of attitude is derived from $\mathbf{R}(t)$ and given at Fig. 7. Time variation of elements of rotation matrix $\mathbf{R}(t)$ is given on Fig. 8. Othogonal properties of the system rotation matrix $\mathbf{R} \in SO(3)$ are exactly preserved by the Lie-group based integration procedure, see Fig. 9, where the matrix entries along the main diagonal as well as the matrix determinant are presented ($\mathbf{R}\mathbf{R}^T = \mathbf{I}$, $\det \mathbf{R} = +1$).

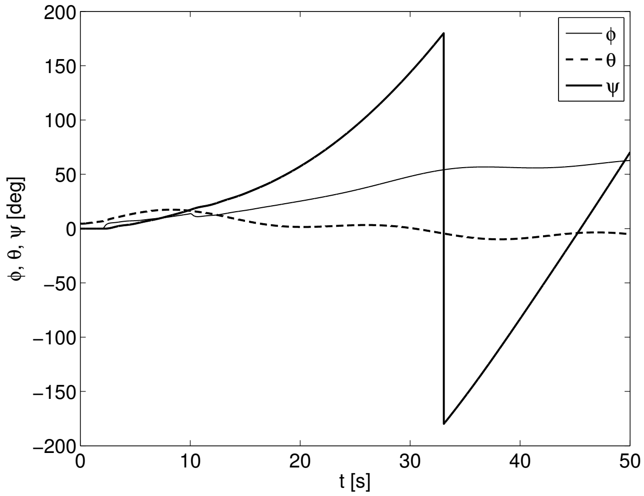

FIGURE 7. ATTITUDE OF THE AIRPLANE Ψ: DE'SPARRE ANGLES – ROLL, PITCH AND YAW ANGLES.

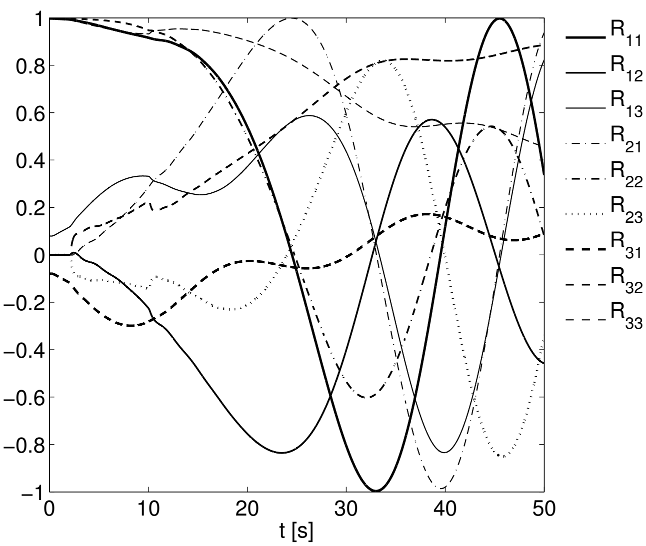

FIGURE 8. ELEMENTS OF ROTATION MATRIX **R**.

CONCLUSION

By integrating aircraft dynamics directly on the state-space manifold, it has been assured "smoothness" of the system rotation response that exhibit no singularities or discontinuities (that would call for re-parameterization of the chosen set of local coordinates) even for large 3D rotation cases. By inspecting integral curves of the aircraft position, angular velocity and elements of rotation matrix (Fig. 3, 5, 6, 8), it is visible that all obtained results are smooth functions without any discontinuities whatsoever. Furthermore, the orthogonal properties of the system ro-

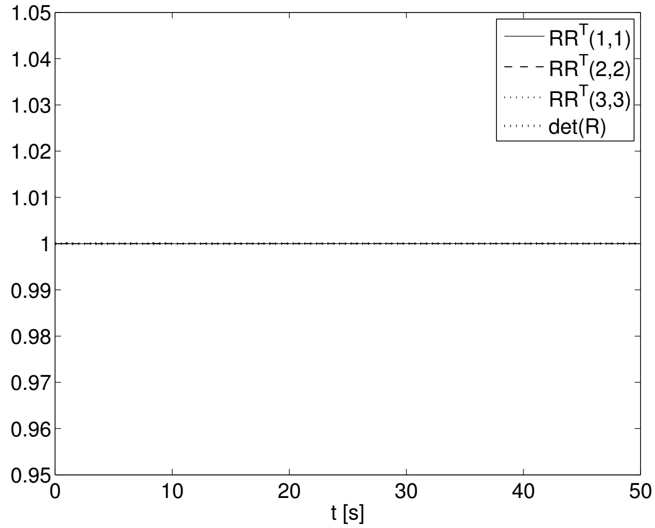

FIGURE 9. PROPERTIES OF ROTATION MATRIX $\mathbf{R}$: DIAGONAL ELEMENTS OF PRODUCT $\mathbf{RR}^T$ AND DETERMINANT $\det \mathbf{R}$.

tation matrix are exactly preserved by the Lie-group integrator (at numerical tolerance), see Fig. 9, where matrix entries along main diagonal as well as matrix determinant are presented. On the contrary, if an integration procedure for the large 3D-rotation-domain simulation cases had been based on the local rotation coordinates such as Euler angles (or any other local coordinates that would lead to "standard" vector-space based integration routine), the discontinuities of the rotation parameters (visible in Fig. 7) would have occurred, meaning that re-parameterization during integration process would have been necessary.

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