DAE Integration Scheme in Lie-Group Settings for Flight Vehicle Forward Dynamics

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Dynamic simulation procedures of flight vehicle maneuvers need robust and efficient integration methods in order to allow for reliable simulation missions. Derivation of such integration schemes in Lie-group settings is especially efficient since the coordinate-free Lie-group dynamical models operate directly on $SO(3)$ rotational matrices and angular velocities, avoiding local rotation parameters and artificial algebraic constraints as well as kinematical differential equations. In the adopted modeling approach, a configuration space of the flight vehicle (modeled as a multibody system comprising k rigid bodies) is modeled as a Lie-group $G = \Re^3 \times ... \times \Re^3 \times SO(3) \times ... \times SO(3)$ (k copies of $\Re^3 \times SO(3)$). The angular velocity of a vehicle element (body i) is given by the left-invariant vector field $\widetilde{\omega}_i \in so(3)$ defined as $\dot{\mathbf{R}}_i(t) = \mathbf{R}_i(t)\widetilde{\omega}_i$ with $so(3)$ being the Lie algebra of $SO(3)$.

To formulate flight vehicle dynamical model in the system state space, modeled also as a Lie-group in the form $\mathbf{S} = \Re^3 \times ... \times \Re^3 \times SO(3) \times ... \times SO(3) \times \Re^3 \times ... \times \Re^3 \times so(3) \times ... \times so(3) \cong TG$, the constrained Boltzmann-Hamel equations are shaped as DAE system of index 1. The system will be integrated by the proposed integration scheme and tested within the framework of several case studies of flight vehicle 3D maneuvers.