On the choice of configuration space for numerical Lie group integration of constrained rigid body systems

Andreas Müller (Institute of Mechatronics, Chemnitz), Zdravko Terze

When a rigid body moves it performs a translation together with a rotation. Moreover a general rigid body motion is a screw motion, i.e. rotation and translations are not independent. Even though standard integration schemes for multibody systems (MBS) neglect the geometry of Euclidean motion in the sense that, within the integration schemes, the position and orientation updates are performed independently. This problem can only be overcome if the Lie group property of rigid body motions is respected. To this end Lie group integration method have been recently applied to MBS that are further subject to additional constraints. In these approaches the direct product Lie group $SO\left(3\right)\times\mathbb{R}^3$ is used as rigid body configuration space. However, three-dimensional Euclidian motions, and thus the motion of a rigid body, are represented by the semidirect product Lie group $SE\left(3\right) = SO\left(3\right) \ltimes \mathbb{R}^3$. Strictly speaking, the direct product $SO\left(3\right) \times \mathbb{R}^3$ can represent the configuration of rigid body but not its motion. The crucial question is whether or not this observation can be carried over to the application of Lie group integration schemes.
In this paper the implications of using the two representations on the performance of Munthe-Kaas integration schemes are investigated. It is pointed out that, although $SE\left(3\right)$ is the only proper representation of rigid body motions, the actual form of the motion equations (using left- or right-invariant, hybrid velocities) also decide about the numerical performance. It is shown that in many cases the $SE\left(3\right)$ representation yields optimal numerical performance for unconstrained as well as for holonomically constrained MBS. The analytic discussion is confirmed by several simple numerical examples.