Angular Momentum Conserving Integration Scheme for Multibody System Dynamics in Lie-Group Setting

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Abstract

In many engineering applications, such as satellite dynamics or various case-studies of the specific locomotion patterns in mechatronics and biomechanics, motion integrals of the system need to be conserved during numerical integration in order to reflect global physical properties of the analysed motion. Derivation of such integration schemes in Lie-group settings should be especially efficient since Lie-group dynamical models operate directly on SO(3) rotational matrices and angular velocities, avoiding local rotation parameters and artificial algebraic constraints as well as kinematical differential equations (differential equations that relate body rotation parameters' derivatives and angular velocity). In this paper, different mathematical models and angular momentum conserving integration schemes, based on Lie-group DAE index 1 formulation of the system dynamics, will be investigated and tested through the several case-studies, such as satellite dynamics with the internal kinematical chain.

In the adopted modeling approach, the configuration space of a MBS comprising k bodies is modeled as a Lie-group $G = \boldsymbol{\mathcal{R}}^3 \times ... \times \boldsymbol{\mathcal{R}}^3 \times SO(3) \times ... \times SO(3)$ (k copies of $\boldsymbol{\mathcal{R}}^3 \times SO(3)$) with the elements of the form $p = (\mathbf{x}_1, ..., \mathbf{x}_k, \mathbf{R}_1, ..., \mathbf{R}_k)$. Each factor $\boldsymbol{\mathcal{R}}^3 \times SO(3)$ represents a configuration of the one single rigid body represented by $(\mathbf{x}_i, \mathbf{R}_i)$ - its position vector and the rotation matrix w.r.t. a global frame (for rigid body i). G is a Lie-group of the dimension $n = 6k$ where k is the number of the rigid bodies.

The angular velocity of a rigid body is given by the left-invariant vector field $\widetilde{\boldsymbol{\omega}}_i \in so(3)$ defined as $\dot{\mathbf{R}}_i(t) = \mathbf{R}_i(t)\widetilde{\boldsymbol{\omega}}_i$ with $so(3)$ being the Lie algebra of $SO(3)$. A velocity of the one body (rigid body i) can thus be represented by the couple $(\mathbf{v}_i, \boldsymbol{\omega}_i) \in \boldsymbol{\mathcal{R}}^3 \times so(3)$.

Aiming on the application of the Lie-group integration scheme proposed in [1], also the MBS state space must be expressed as a Lie-group. Therefore, the MBS state space $\boldsymbol{\mathcal{S}} = \boldsymbol{\mathcal{R}}^3 \times ... \times \boldsymbol{\mathcal{R}}^3 \times SO(3) \times ... SO(3) \times \boldsymbol{\mathcal{R}}^3 \times ... \times \boldsymbol{\mathcal{R}}^3 \times so(3) \times ... \times so(3) \widetilde{=} TG$ is introduced, with the elements $q = (\mathbf{x}_1, ..., \mathbf{x}_k, \mathbf{R}_1, ..., \mathbf{R}_k, \mathbf{v}_1, ..., \mathbf{v}_k, \boldsymbol{\omega}_1, ..., \boldsymbol{\omega}_k)$. This is a Lie-group itself and possess the Lie-algebra $\boldsymbol{s} = \boldsymbol{\mathcal{R}}^3 \times ... \times \boldsymbol{\mathcal{R}}^3 \times so(3) \times ... \times so(3) \times \boldsymbol{\mathcal{R}}^3 \times ... \times \boldsymbol{\mathcal{R}}^3 \times \boldsymbol{\mathcal{R}}^3 \times ... \times \boldsymbol{\mathcal{R}}^3$ with the element $z = (\mathbf{v}_1, ..., \mathbf{v}_k, \widetilde{\boldsymbol{\omega}_1}, ..., \widetilde{\boldsymbol{\omega}_k}, \dot{\mathbf{v}}_1, ..., \dot{\mathbf{v}}_k, \dot{\boldsymbol{\omega}}_1, ..., \dot{\boldsymbol{\omega}}_k)$. Furthermore, the operations on the Lie-group $\boldsymbol{\mathcal{S}}$ and its Lie-algebra $\boldsymbol{s}$ (such as product in $\boldsymbol{\mathcal{R}}^3 \times SO(3)$), addition in $\boldsymbol{\mathcal{R}}^3 \times so(3)$, multiplication by scalar in $\boldsymbol{\mathcal{R}}^3 \times so(3)$, exponential map in $\boldsymbol{\mathcal{R}}^3 \times so(3)$ and bracket in $\boldsymbol{\mathcal{R}}^3 \times so(3)$ can be introduced [1], allowing synthesis of the subsequent integration routines. To formulate dynamical model of the constrained MBS in the introduced state space, we start from the constrained Boltzmann-Hamel equations

$$\mathbf{M}(p)\dot{\mathbf{v}} + \mathbf{C}^T(p)\boldsymbol{\lambda} = \mathbf{Q}(p, \mathbf{v}, t)$$

$$\dot{p} = p \cdot \widetilde{\mathbf{v}} \tag{1}$$

$$\boldsymbol{\Phi}(p) = \mathbf{0},$$

where $\mathbf{M}$ is $n \times n$ dimensional generalized inertia matrix, $\mathbf{v} \in \boldsymbol{\mathcal{R}}^n$, $\mathbf{v} = [\mathbf{v}_1, ..., \mathbf{v}_k, \boldsymbol{\omega}_1, ..., \boldsymbol{\omega}_k]^T$ are the system velocities (k bodies are assumed), $\mathbf{Q}$ represents the external and all other forces, $\boldsymbol{\lambda} \in \boldsymbol{\mathcal{R}}^m$ is the vector of Lagrange multipliers and $\mathbf{C}$ is $m \times n$ dimensional constraint Jacobian, such that $\boldsymbol{\Phi}'(p) \cdot \widetilde{\mathbf{v}} = \mathbf{C}(p)\mathbf{v}$, where $\boldsymbol{\Phi}'$ is the differential mapping of the constraint mapping $\boldsymbol{\Phi} : G \to \boldsymbol{\mathcal{R}}^m$. Consequently, a MBS is constrained to evolve on the n-m dimensional sub-manifold $S = \{p \in G : \boldsymbol{\Phi}(p) = 0\}$. The equation $\dot{p} = p \cdot \widetilde{\mathbf{v}}$ in (1) achieves the reconstruction of the motion from the velocities $\mathbf{v}$.

The system (1) is a DAE system of index 3. Within the framework of this work, the equation (1) will be re-shaped into the DAE of index 1 form by including the kinematical constraints at the acceleration level $\ddot{\boldsymbol{\Phi}}(p, \mathbf{v}, \dot{\mathbf{v}}) = 0$ (instead of $\boldsymbol{\Phi}(p) = 0$) and integrated by the integration algorithms based on the state space formulation [1]. During integration, a conservation of the system angular momentum will be 'controlled' by the several algorithms that will be tested within the integration procedures.

The first algorithm will be based on introduction of the additional non-holonomic constraint equation, expressing conservation of the system angular momentum, during integration of DAE-index-1 [1]. Within this procedure, a conservation property of the algorithm will be assured by using projective velocities' partitioning criterion [2], adapted to be valid on TG. Since the stabilization algorithm [2] takes into account geometrical properties of the state space that is influenced by inertial characteristics of the system (i.e. TG geometric properties are influenced also by the system kinetics, and not only by kinematics), it is expected that the designed projective procedure will establish efficient conservative integration scheme, compatible also with higher order integration methods. The simulation results will be compared with the angular momentum conserving integration schemes that will be constructed on the basis of Newmark family type of algorithms [3], [4], [5], formulated for DAE-index-1 mathematical framework in Lie-group settings.

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References

[1] Z. Terze, A. Müller, D. Zlatar. Lie-group integration method for constrained multibody systems in stabilized DAE-index-1 form. *Multibody System Dynamics*, 2012 (submitted).

[2] Z. Terze and J. Naudet. Geometric properties of projective constraint violation stabilization method for generally constrained multibody systems on manifolds. *Multibody System Dynamics*, 20:85-106, 2008.

[3] J. C. Simo and K. K. Wong. Unconditionally stable algorithms for rigid body dynamics that exactly preserve energy and momentum. *Int. J. Numer. Meth. Engng.*, 31:19-52, 1991.

[4] P. Krysl. Explicit momentum-conserving integrator for dynamics of rigid bodies approximating the midpoint Lie algorithm. *Int. J. Numer. Meth. Engng.*, 63:2171-2193, 2005.

[5] P. Krysl, L. Endres. Explicit Newmark/Verlet algorithm for time integration of the rotational dynamics of rigid bodies. *Int. J. Numer. Meth. Engng.*, 62:2154-2177, 2005.