Dynamic Simulation of Helicopter 3D Airborne Maneuvers with Numerical Integration Scheme in Lie-Group Setting

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Abstract

Dynamic simulation procedures of the aircraft 3D motion need robust and efficient integration methods in order to allow for reliable (and possibly real-time) simulation missions. Derivation of such integration schemes in coordinate-free Lie-group settings is especially efficient since Lie-group dynamical models operate directly on $SO(3)$ rotational matrices and angular velocities, avoiding local rotation parameters and artificial algebraic constraints as well as kinematical differential equations. These integration features of the coordinate-free formulations should be especially beneficial for the flight vehicle simulation missions since a realization of the aircraft complex 3D maneuvers often requires numerical forward dynamics that includes complete 3D rotation domain. In such cases, the utilization of the 'standard' vector-space-based modeling procedures (with the local rotation parameters) leads toward kinematical singularities and re-parameterization of the rotation domain, which requires further computational burden. Along this line, a numerical integration scheme in Lie-group settings for the helicopter forward dynamics as well as inverse dynamics control problem is presented and discussed in the paper.

In the adopted modeling approach, the configuration space of the helicopter 6 DOF model is introduced as a six-dimensional Lie-group $G = \mathcal{R}^3 \times SO(3)$ with the element of the form $p = (\mathbf{x}, \mathbf{R})$ that consists of the helicopter body mass center position vector $\mathbf{x}$ and the body attitude, given by the rotation matrix $\mathbf{R}$ w.r.t. the global frame.

The body angular velocity is given by the left-invariant vector field $\widetilde{\boldsymbol{\omega}} \in so(3)$ defined as $\dot{\mathbf{R}}(t) = \mathbf{R}(t)\widetilde{\boldsymbol{\omega}}$ with $so(3)$ being the Lie algebra of $SO(3)$. A velocity of the helicopter can thus be represented by the couple $(\mathbf{v}, \boldsymbol{\omega}) \in \mathcal{R}^3 \times so(3)$, where $\mathbf{v}$ is the body mass center velocity.

Aiming on the application of the Lie-group integration scheme proposed in [1], also the vehicle state space must be expressed as a Lie-group. Therefore, the helicopter state space $\boldsymbol{\mathcal{S}} = \mathcal{R}^3 \times SO(3) \times \mathcal{R}^3 \times so(3) \cong TG$ is introduced, with the element $q = (\mathbf{x}, \mathbf{R}, \mathbf{v}, \boldsymbol{\omega})$. This is a Lie-group itself and possess the Lie-algebra $\boldsymbol{s} = \mathcal{R}^3 \times so(3) \times \mathcal{R}^3 \times \mathcal{R}^3$ with the element $z = (\mathbf{v}, \widetilde{\boldsymbol{\omega}}, \dot{\mathbf{v}}, \dot{\boldsymbol{\omega}})$. Furthermore, we introduce the operations on the Lie-group $\boldsymbol{\mathcal{S}}$ and its Lie-algebra $\boldsymbol{s}$ as follows:
Product in $\mathcal{R}^3 \times SO(3) : (a, b, c, d) \cdot (e, f, g, h) = (a + e, b \cdot f, c + g, d + h)$.
Addition in $\mathcal{R}^3 \times so(3) : (v, w, c, d) + (\bar{v}, \bar{w}, \bar{c}, \bar{d}) = (v + \bar{v}, w + \bar{w}, c + \bar{c}, d + \bar{d})$.
Multiplication by scalar in $\mathcal{R}^3 \times so(3) : \alpha(v, w, c, d) = (\alpha v, \alpha w, \alpha c, \alpha d)$.
Exponential map in $\mathcal{R}^3 \times so(3) : \exp(v, w, c, d) = (v, \exp(w), c, d)$.
Bracket in $\mathcal{R}^3 \times so(3) : \left[(v, w, c, d), (\bar{v}, \bar{w}, \bar{c}, \bar{d})\right] = (0, w \times \bar{w}, 0, 0)$.

Here, on the right hand side of definitions, '$\cdot$' is the multiplication in $SO(3)$, '$+$' is the addition in $\mathcal{R}^3$ and $so(3)$ and $\exp$ is the exponential map on $so(3)$.

The dynamical model of the airborne helicopter, modeled as an under-actuated mechanical system in the introduced Lie-group state space, can be formulated as

$$\begin{aligned} \mathbf{M}(p)\dot{\mathbf{v}} &= \mathbf{Q}(p, \mathbf{v}, t) + \mathbf{B}^T(p)\mathbf{u} \\ \dot{p} &= p \cdot \widetilde{\mathbf{v}}, \end{aligned} \tag{1}$$

where $\mathbf{M}$ is 6×6 dimensional generalized inertia matrix, $\mathbf{v} \in \mathcal{R}^6$, $\mathbf{v} = [\mathbf{v}, \boldsymbol{\omega}]^T$ are the system velocities, $\mathbf{Q}$ stands for the external and non-linear velocity forces and $\mathbf{B}^T\mathbf{u}$ represents helicopter actuation (4×6 matrix $\mathbf{B}$ standing for the influence of the pilot control inputs $\mathbf{u}$ on the generalized actuation forces). For the specified control inputs, forward dynamics of the helicopter airborne movement can be obtained via Lie-group integration scheme proposed in [1] and furtherly commented in [2]. During integration, the helicopter 3D motion can be numerically reconstructed from the vehicle velocity field $\mathbf{v}$, by using the equation $\dot{p} = p \cdot \widetilde{\mathbf{v}}$ in (1).

Beside forward dynamics problem in Lie-group setting as explained above, the paper will also address inverse dynamics control problem [3] of the helicopter airborne motion. With this aim in view, aircraft 3D trajectories will be specified by

imposing system algebraic (control) constraints in the form

$$\mathbf{\Phi}(p) - \psi(t) = \mathbf{0}, \tag{2}$$

that can be also given on the acceleration constraint level

$$\mathbf{C}(p)\dot{\mathbf{v}} - \vartheta(p, \mathbf{v}, t) = \mathbf{0}. \tag{3}$$

In (2) and (3) $\mathbf{C}$ is 4×6 dimensional constraint matrix, such that $\mathbf{\Phi}'(p) \cdot \widetilde{\mathbf{v}} = \mathbf{C}(p)\mathbf{v}$, where $\mathbf{\Phi}'$ is the differential mapping of the constraint mapping $\mathbf{\Phi} : G \to \boldsymbol{\mathcal{R}}^4$, and $\psi(t)$ is the additional rheonomic term. By putting together (1) and (2) or (3), the helicopter inverse dynamics control problem can be formulated as DAE integration problem [4] in Lie-group settings. Assuming flatness of the under-actuated system [5], the helicopter state space variables $q = (\mathbf{x}, \mathbf{R}, \mathbf{v}, \boldsymbol{\omega})$ as well as control inputs $\mathbf{u}$ can be determined by the integration algorithm based on the geometric concepts [6], [7], [1], [2]. During integration, DAE hidden constraints may be stabilized via Lie-group stabilization algorithms described in [1] and [2].

In the paper, the outlined computational procedure will be presented in detail. The numerical algorithm will be demonstrated and tested within the framework of the several case-studies with the specified helicopter 3D maneuvers.

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